"That" as a relative clause works as a direct object and as a subject of the verb

https://forum.wordreference.com/threads/when-and-when-not-to-use-that.1350280/

You can drop "that" when it introduces or works as the direct object, but you can't do it if it works as the subject of the verb.

I thought (that) I was too shy (<u>What did you think? (That) I was too shy</u>) In this case, "I" is the subject of "thought", while "that" is a conjunction that introduces a subordinate clause that works as the direct object of the verb. You can drop "that".

The car (that) I bought is red (What did you buy? The car and it is red) In this sentence, we have a relative clause in which "I" is the subject of "bought", and "that" is a relative pronoun that is the direct object of bought. Therefore, you can drop it too.

The car <u>that</u> is parked outside is red. (<u>Restrictive</u>, only that one you can replace it by which between commas: the car, which is park outside, is red) In this case, there is another relative clause, but "that" works as the subject of "is", so you can't drop it.

To see restrictive and non-restrictive *which* and *that*: https://grammar.yourdictionary.com/grammar-rules-and-tips/when-to-use-which-or-that.html

But there is much more about this topic because you can use *that* with bridge and non-bridge verbs, nouns and adjectives. There are <u>three pages</u> on this on https://www.quickanddirtytips.com/education/grammar/when-to-delete-that?page=2